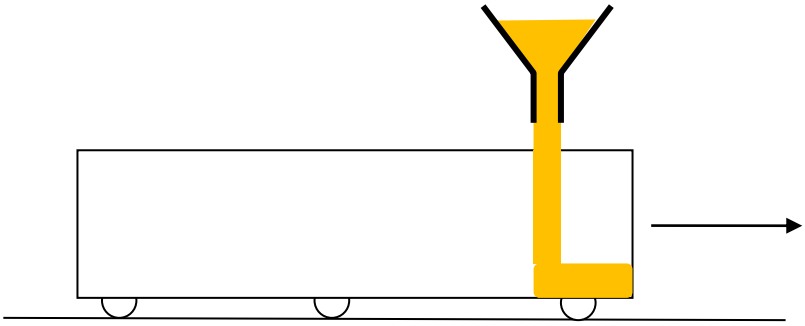
Teacher notes
Topic A

The leaking carriage

In a previous note we examined a problem where mass changes. The gravel falling on the carriage increases the mass on the carriage at a rate of μ kg per second. The carriage is moving at constant speed v.



The acceleration is zero so we would expect a zero force acting on the belt. But this is not correct because the mass is changing. We need to use $F_{\text{net}} = \dfrac{\Delta p}{\Delta t}$ and apply it to the system of the carriage and all the gravel (including that in the hopper). At t = 0 the momentum of the carriage and the gravel in it is $p = Mv$. After time Δt, the mass on the carriage will increase by $\Delta m = \mu \Delta t$ and so the momentum will be $p' = (M + \Delta m)v = (M + \mu \Delta t)v$. The *change* in momentum is thus $\Delta p = v\mu \Delta t$ and so the force is $F_{\text{net}} = \dfrac{v\mu \Delta t}{\Delta t} = \mu v$.

So we have a situation where a force is required to keep the belt moving at constant speed.

Now suppose we look at the reverse problem, namely a carriage filled with gravel is leaking gravel at a rate of μ kg per second. What force is required to keep the carriage moving at constant speed?

The initial momentum is $p = Mv$. After time Δt, a mass Δm left the carriage and so the momentum is $p' = (M - \Delta m)v + \Delta mv = Mv$. Hence, $\Delta p = 0$ and so $F = 0$. This is because the leaking gravel still has speed v to the right as it exits the carriage.

This is somewhat counterintuitive. Why are the two situations of gravel filling a carriage and gravel leaving a carriage different?

In the case of the carriage being filled, the gravel falls with zero horizontal velocity whereas in the case of the leaking carriage the gravel has the same horizontal velocity before and after leaving the carriage. This means that in the case of filing the carriage a force is necessary to accelerate the gravel from zero velocity to the velocity of the carriage.